

## 11-4 Geometric Series

sum of a geometric sequence

$$2 + 6 + 18 + 54 + 162$$

$$r = 3$$

$$S_5 = 2 + 6 + 18 + 54 + 162$$

$$-3 \cdot S_5 = -6 - 18 - 54 - 162 - 486$$

$$-2S_5 = 2 - 486$$

$$S_5 = \frac{2 - 486}{-2} = 242$$

$$S_n = \frac{a_1 - r \cdot a_n}{1 - r}$$

$$S_n = \frac{a_1 - r a_n}{1 - r} \quad \leftarrow a_n = a_1 r^{n-1} \quad r \neq 1$$

$$\frac{a_1 - r \cdot a_1 r^{n-1}}{1 - r} \quad r^1 \cdot r^{n-1}$$

$$S_n = \frac{a_1 - a_1 r^n}{1 - r}$$

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$$S_n = \frac{a_1 - a_1 r^n}{1 - r} \quad r \neq 1$$

$$S_n = \frac{a_1(1 - r^n)}{1 - r}$$

Ex 1:  
Find the sum.  
 $a_1 = 7776$   
 $a_n = 6$

$$r = \frac{-1}{6}$$

$$S_n = \frac{a_1 - r \cdot a_n}{1 - r}$$

$$= \frac{7776 + \left(\frac{1}{6}\right)6}{1 + \left(\frac{1}{6}\right)}$$

$$= \frac{7777}{\frac{7}{6}}$$

$$S_n = 6,666$$

Ex 2:

$a_1 = ?$   
 $S_8 = 765$

$r = 2$

$$S_n = \frac{a_1(1 - r^n)}{1 - r}$$

$$765 = \frac{a_1(1 - 2^8)}{1 - 2}$$

$$765 = \frac{a_1(1 - 256)}{-1}$$

$$-765 = a_1(-255)$$

$$3 = a_1$$

Sigma Notation

$$\sum_{n=1}^{12} 3 \cdot 2^{n-1}$$

Expanded Form

$$= 3 + 6 + 12 + \dots + 6144$$

$$S_{12} = \frac{3 - 2 \cdot 6144}{1 - 2} = 12,285$$

Sigma Notation

$$\sum_{n=1}^5 6 \cdot \left(\frac{1}{3}\right)^{n-1} = 6 + 2 + \frac{2}{3} + \frac{2}{9} + \frac{2}{27}$$

Expanded Form

Put the following series into sigma notation.

$$48 + 24 + 12 + 6 + \dots + \frac{3}{8}$$

$$a_n = 48 \left(\frac{1}{2}\right)^{n-1}$$

Need  
eqn. for an

$$\sum_{n=1}^8 48 \left(\frac{1}{2}\right)^{n-1}$$

$$\begin{aligned} \frac{3}{8} &= 48 \left(\frac{1}{2}\right)^{n-1} \\ \frac{1}{128} &= \frac{1}{2}^{n-1} \\ \left(\frac{1}{2}\right)^7 &= \frac{1}{2}^{n-1} \\ 7 &= n-1 \\ 8 &= n \end{aligned}$$

29. **HEALTH** Contagious diseases can spread very quickly. Suppose five people are ill during the first week of an epidemic and that each person who is ill spreads the disease to four people by the end of the next week. By the end of the tenth week of the epidemic, how many people have been affected by the illness?

HW

p597

15-27 odd, 30, 33, 35, 39, 41, 43, 47